Nonlinear Inversion from Nonlinear Filters for Ocean Acoustics

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Award # N00014-05-1-0087

LONG TERM GOALS

The long term goals of this research are to develop practical and efficient algorithms for application the nonlinear inversion problems encountered in ocean acoustics. Such algorithms would be used for estimating or accounting for the effects of the environment on acoustic propagation, detection and tracking in shallow water.

OBJECTIVES

The specific objectives of this research are adapt a specific nonlinear filter, known as a Daum filter, for acoustic inversion of shallow water environmental properties, and to assess the performance of this nonlinear filter relative to local linear inversion on the one hand and global methods, e.g. Monte Carl methods on the other hand.

APPROACH

Many inverse problems of interest in ocean acoustics are intrinsically nonlinear, e.g. inverting measured pressure data for bottom and scattering properties. The solution to the nonlinear inversion problem is usually approached in one of two ways. The first way is to assume a starting model, which one hopes is near to the true model, then recursively solve a linearized version of the inverse problem for corrections to the starting model and model covariance. The advantage of this approach is that the numerical implementation of the solution algorithm is relatively straightforward and in a linear problem the statistical properties are well defined and will remain gaussian if they start out gaussian. However linearization of a nonlinear system can produce biased estimates for two reasons: 1. Linearization of the system and/or measurement equations may not be a good approximation, and 2. Nonlinear systems do not maintain gaussian statistics as they evolve even if they are initially gaussian. Another problem with linearizing a nonlinear system is that with a poor starting guess the solution algorithm may never converge to the true answer. If the starting model represents a point near a local minimum of the solution space, the final solution will be trapped in that local minimum, and never converge to the true answer. This can be circumvented by using Monte Carlo techniques to randomly sample the solution space for starting models.

The other class of solution methods attack the nonlinear problem directly by using simulated annealing or genetic algorithms. The disadvantage of these directly nonlinear methods, is that there is

Report Documentation Page

Form Approved
OMB No. 0704-0188

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1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
30 SEP 2005		

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Nonlinear Inversion from Nonlinear Filters for Ocean Acoustics	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
University of Washington,Applied Physics Laboratory,College of Ocean and Fisheries Sciences, 1013 N.E. 40th Street,Seattle,WA, 98195	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES
The original document contains color images.

14. ABSTRACT
The long term goals of this research are to develop practical and efficient algorithms for application the nonlinear inversion problems encountered in ocean acoustics. Such algorithms would be used for estimating or accounting for the effects of the environment on acoustic propagation, detection and tracking in shallow water.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**		**6**	

no way to conveniently propagate the statistical properties of the solution through to the final result. One solution to this problem is to find the global minimum in the solution space, if one exists, then linearize about the solution representing the global minimum and do a statistical analysis about that solution. This was done by Potty et al.(2000), who employed a genetic algorithm followed by linear analysis about the solution determined by the genetic algorithm.

The recursive algorithms commonly employed for the estimation of the model and covariance relative to some initial starting values bear a strong resemblance to Kalman filters, which are commonly employed in target tracking algorithms. The original Kalman filter was derived for strictly linear systems. However, the Extended Kalman Filter can be applied to systems which are weakly nonlinear. In the late 1980s Frederick Daum, a mathematician working at Raytheon Corporation, developed a fully nonlinear formulation to the filtering problem for target tracking (Daum, 1985, 1986, 1987). His theory is elegant, but impractical from an implementation point of view. Sometime later Schmidt (Schmidt, 1993) succeeded in deriving an approximate algorithm based on Daum's original theory, and developed a successful numerical implementation of a nonlinear filter that was a significant improvement to the Kalman and Extended Kalman filters for the type of tracking problem Schmidt was interested in.

This research aims to develop an ocean acoustic inversion algorithm based on Schmidt's (1993) implementation of Daum's nonlinear filtering theory. The purpose is to be able to carry along the statistics of the geoacoustic model parameters through the inversion process. The work is much more than a straightforward "rename the variables and code it up". However, the tracking algorithms do bear resemblance to the iterative inversion algorithms for updating model parameter means and covariances of an iterated inverse problem as in, for example, Menke (1983). Most estimation problems can be cast into an interative form, whereby the state vector, which in our case is the ocean bottom model vector, is updated sequentially as data is added. Estimation filter formulations are also natural for range dependent or time dependent environments. Daum's original theory and Schmidt's practical implementation assumes nonlinear dynamics and a linear relationship between the measurement and state vectors. In our case the measurement vector, complex pressure say, and the state vector, the bottom model, are not linearly related. The filter needs to be re-derived from scratch with the measurement to state vector relationship appropriate for our ocean acoustic application. Once re-derived, it will need to be coded, and checked against results for linear inverse problems. Dosso (e.g. Dosso and Wilmut, 2002) at the University of Victoria has developed a Monte Carlo inversion method for the ocean acoustics problem. This is computationally very intensive, but he does get the full probability density function (pdf) for the model parameters. Because the Schmidt implementation of the Daum theory propagates the additional terms in the mean and covariance of the state vector pdf, it falls in between the standard linear inversion methods and Dosso's Bayesian Monte Carlo methods.

Currently employed algorithms for nonlinear problems such as simulated annealing and genetic algorithms have no mechanism for propagating the statistics. What the nonlinear filter algorithm will do is provide a natural mechanism for updating the statistics as a solution is determined. A comparison of the filter with an algorithm such as simulated annealing would be illuminating, and a valuable check on the filter algorithm itself.

Filter type algorithms are ideally suited to inverse problems with time dependent oceanography or range dependence. We do not anticipate attempting to include time dependent oceanography at this time, but we would like to look at the issue of range dependent inversion. The idea would be to sequentially update parameter estimates as a function of range. Also note that any inversion algorithm

can be cast into a filter like algorithm by supplying the data sequentially and updating the model parameter estimates sequentially as data is added to the problem, or a smoother by considering the complete data set, and working both forwards and backwards through the data set. In the end, probably the best formulation to use for a given inverse problem depends on the noise statistics. This is also something we propose to investigate.

Linear inverse problems admit the construction of both data and model resolution matrices. These resolution matrices can be used as metrics with which to estimate model uniqueness and data predictability. We will be able to construct resolution matrices for the nonlinear problem and compare them with their fully linear equivalents.

WORK COMPLETED

Previously we have taken a deterministic view of the resolution matrix. When noise or incomplete measurements are included, a statistical view must be adopted. The well known Fisher information matrix is directly related to the model resolution matrix. The Fisher information matrix is $\mathbf{F} = \mathbf{G}^{-g}\mathbf{E}^{-1}\mathbf{G}$, and the resolution matrix is $\mathbf{R} = \mathbf{AF}$, where $\mathbf{E}$ is the covariance of the measurements plus the error in the forward model, $\mathbf{A}$ is the covariance of the model estimate after incorporation of the measurement – the *a posteriori* covariance, $\mathbf{G}$ is the matrix of model sensitivities, the partial derivatives w.r.t. the model parameters, and $-g$ denotes the generalized inverse of $\mathbf{G}$. The trace of the resolution matrix is simply related to the total number of model parameters of the *a priori* and *a posteriori* covariances. In the next work period will be exploring these relationships numerically and developing relations for a nonlinear version of the resolution matrix.

As stated above an inverse problem can be recast as a filtering problem. A strictly linear problem becomes a Kalman Filter (KF). A problem that has been locally linearized becomes an Extended Kalman Filter (EKF), and the fully nonlinear problem can be represented as a Daum – Schmidt Filter. We have completed working computer code for a simple EKF with which to compare more complicated inverse models.

RESULTS

Figure 1 shows numerical results for the comparison of an EKF (white) and gradient search (black) applied to a weakly nonlinear test problem. The location of a stationary source is solved for using 10 receivers each with 10 noisy data points. Each method starts from some initial guess and has a 95% confidence ellipse about its final point. The colorful surface is the objective surface. The red square, which is the solution point, is not exactly at the minimum of the surface. The data have additive Gaussian noise, which slightly biases the surface. Regularization of the gradient descent was done by truncating the small eigenvalues. We developed the EKF code as this will be the algorithm against which we will compare the Daum Filter.

Figure 2 shows the convergence of the location as more receivers, each with 10 data points are incorporated into the source location estimate.

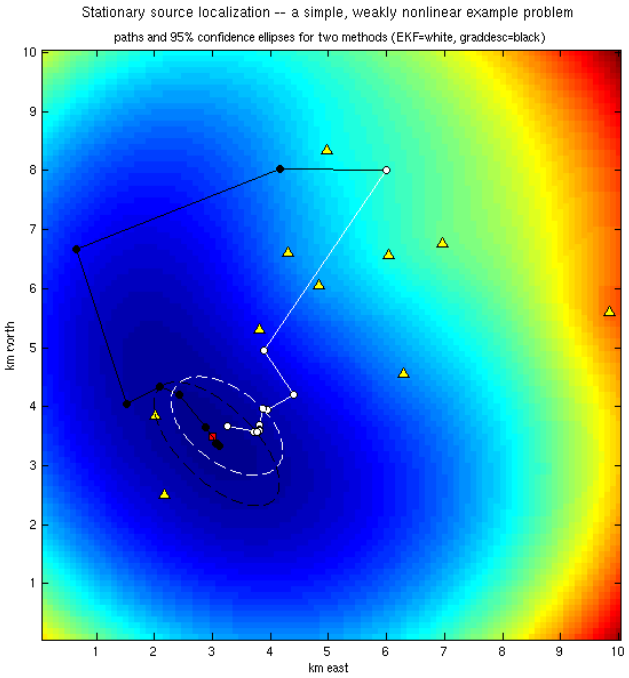

Figure 1. ***Comparison of gradient descent (black) with an EKF (white) for a slightly nonlinear source localization problem. The yellow triangles are the receiver locations, each with 10 data points. Noise in the measurements causes the bias in the final location.***

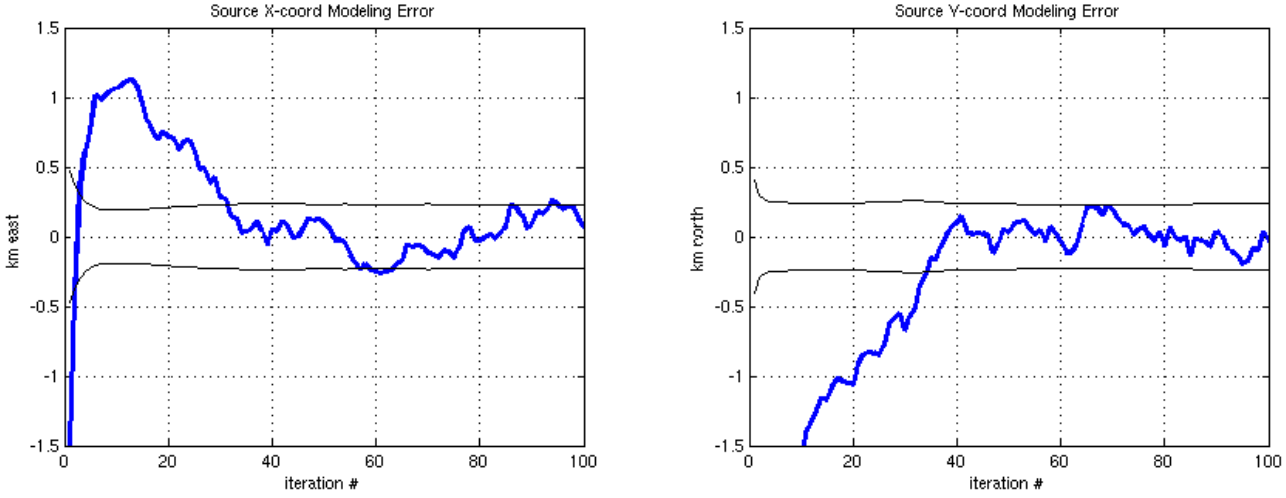

Figure 2. ***Model error for the example in Figure1. As more receivers, each with 10 data points with noise, are included, the final source location is estimated.***

IMPACT/APPLICATIONS

A nonlinear, well characterized filter-based inversion method and algorithm will have application to environmental estimation and target tracking. A practical method way to compute the resolution for a nonlinear inversion will have an impact on the characterization of uncertainty and uniqueness of environmental estimates required for acoustic propagation.

RELATED PROJECTS

Our research is directly related to other programs studying effects of uncertainty in the environment, measurements, and models on acoustic propagation, and target detection and characterization.

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